Monte Carlo Simulation

Rayleigh and Raman Scattering

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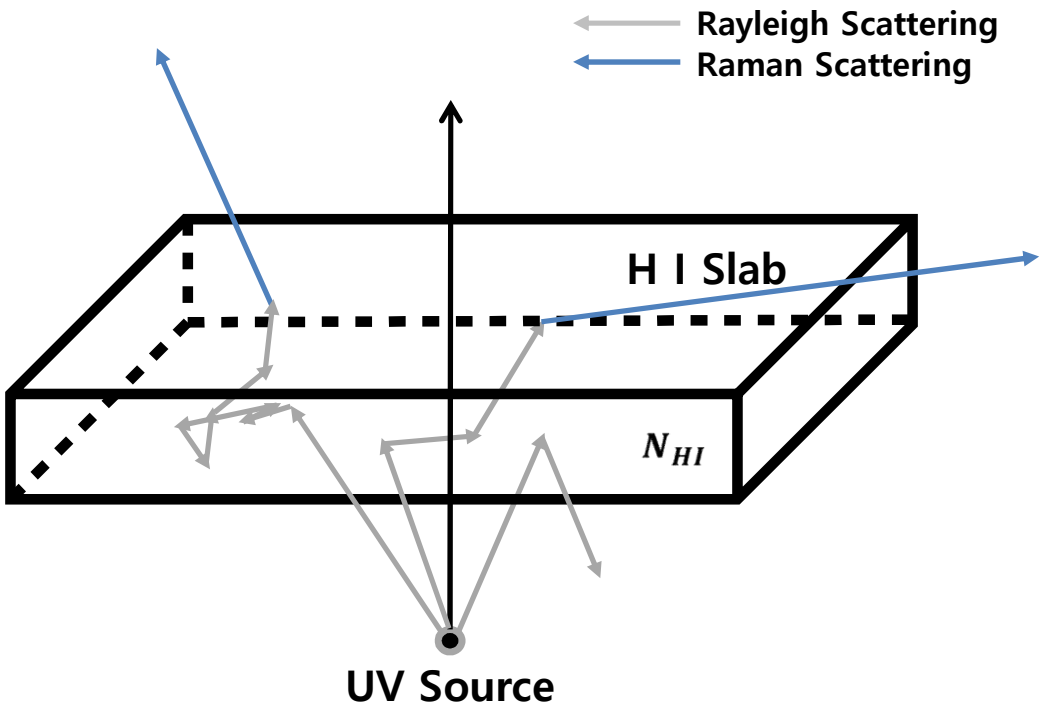
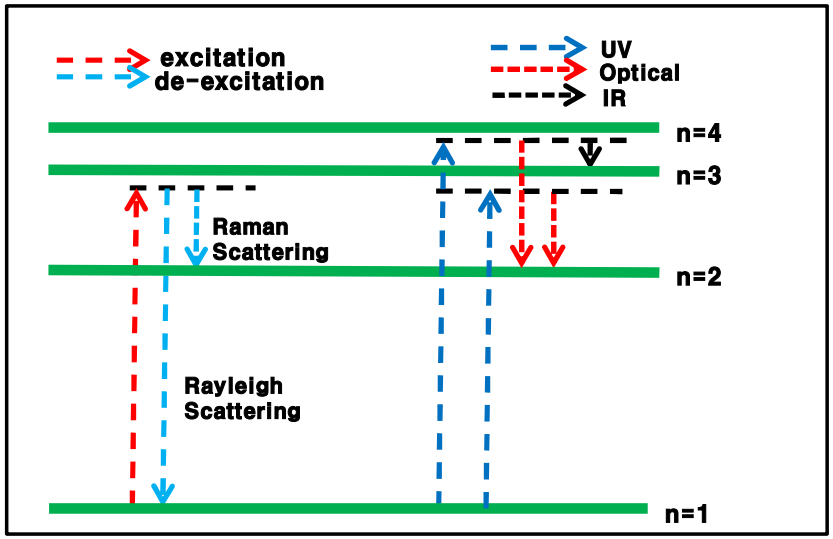
Introduction



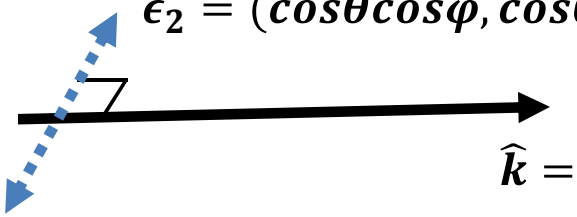
Geometry



Hydrogen Atomic Level

Density Matrix Transition

$$\overrightarrow{\epsilon_1} = (-sin\varphi, cos\varphi, 0)$$
$$\overrightarrow{\epsilon_2} = (cos\theta cos\varphi, cos\theta sin\varphi, -sin\theta)$$



$$\hat{k} = (sin\theta cos\varphi, sin\theta sin\varphi, cos\theta)$$

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} \frac{I+Q}{2} & \frac{U+iV}{2} \\ \frac{U-iV}{2} & \frac{I-Q}{2} \end{pmatrix}$$

Wavevector and Basises of Polarization Vector

Density Matrix

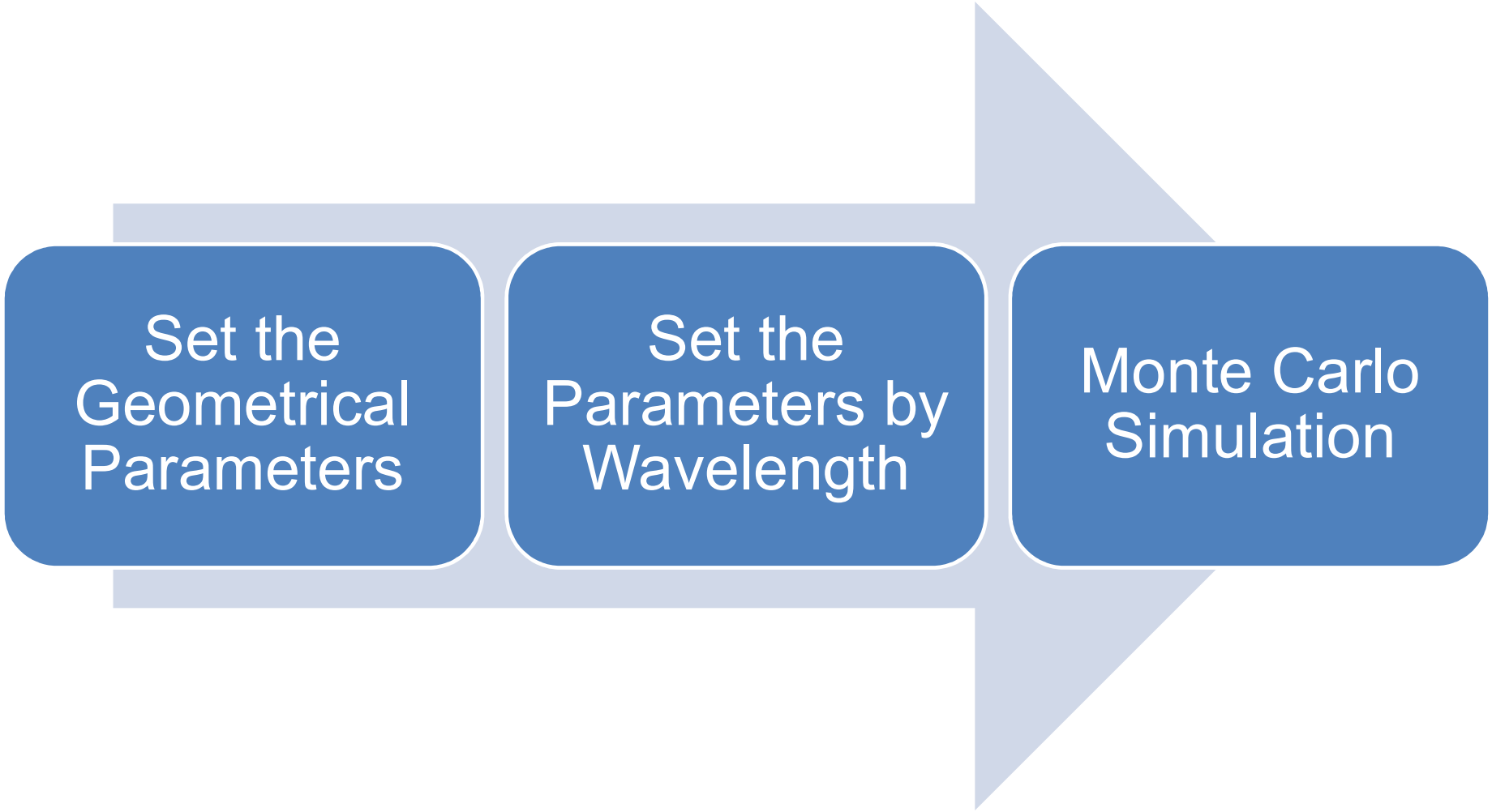
$$\rho'_{11} = \left(co\ \ ^2\Delta\varphi\right)\rho_{11} - (cos\theta sin2\Delta\varphi)\rho_{12} + \left(si\ \ ^2\Delta\varphi cos^2\theta\right)\rho_{22}$$

$$\rho'_{22} = \left(cos^2\theta' sin^2\Delta\varphi\right)\rho_{11} + cos\theta'(2sin\theta sin\theta' sin\Delta\varphi + cos\theta cos\theta' sin2\Delta\varphi)\rho_{12} + (cos\theta cos\theta' cos\Delta\varphi + sin\theta sin\theta')\rho_{22}$$

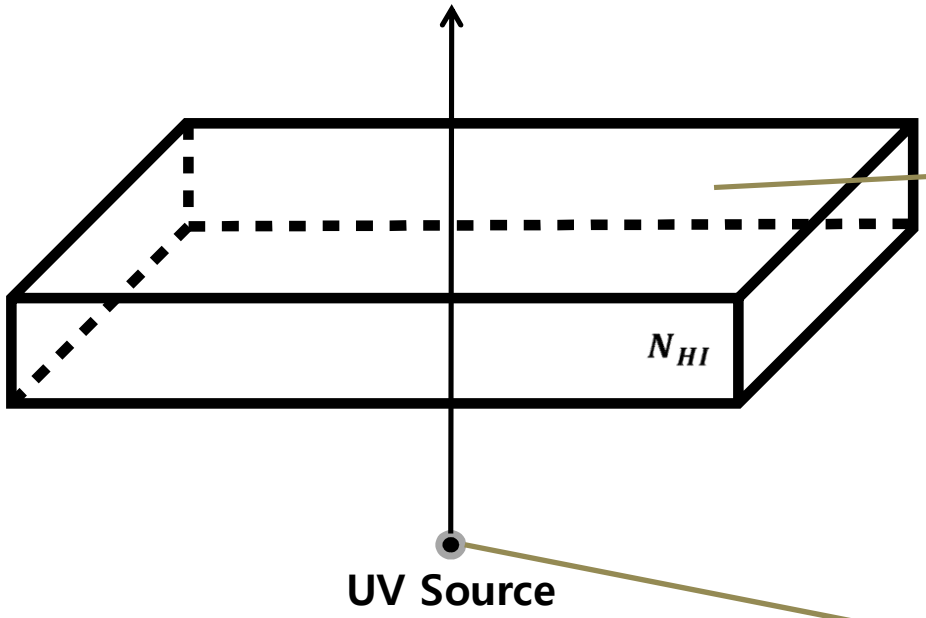
$$\rho'_{12} = (1/2cos\theta' sin2\Delta\varphi)\rho_{11} + (cos\theta cos\theta' cos2\Delta\varphi + sin\theta sin\theta' cos\Delta\varphi)\rho_{12} - cos\theta(sin\theta sin\theta' sin\Delta\varphi + 1/2cos\theta cos\theta' sin2\Delta\varphi)\rho_{22}$$

Density Matrix Formula for Scattered Photon
(Ahn and Lee 2015, Chang et al. 2015)

Process


Set the Geometrical Parameters
Set the Parameters by Wavelength
Monte Carlo Simulation

Set the Geometrical Parameters



$$N_{HI} = n_{HI} d$$

N_{HI} : Column density (cm^{-2})

n_{HI} : Number density (cm^{-3})

d : Physical distance (cm)

$\lambda_{max}, \lambda_{min}$: Maximum and minimum wavelength

f_{λ} : The number of input photons

Set the Parameters by Wavelength

Subroutine SIGMA

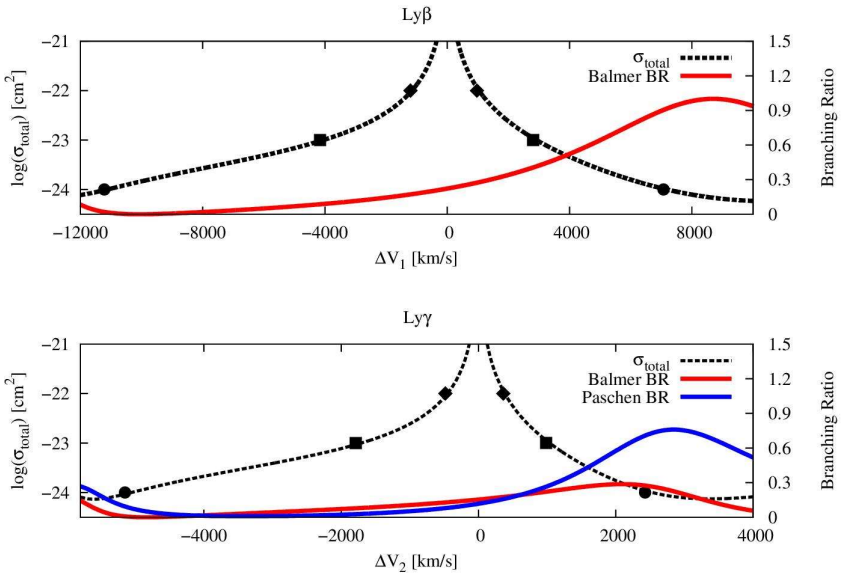
$\sigma_{total}(\lambda)$: Total cross section

$BR_2(\lambda)$: Branching ratio to level 2

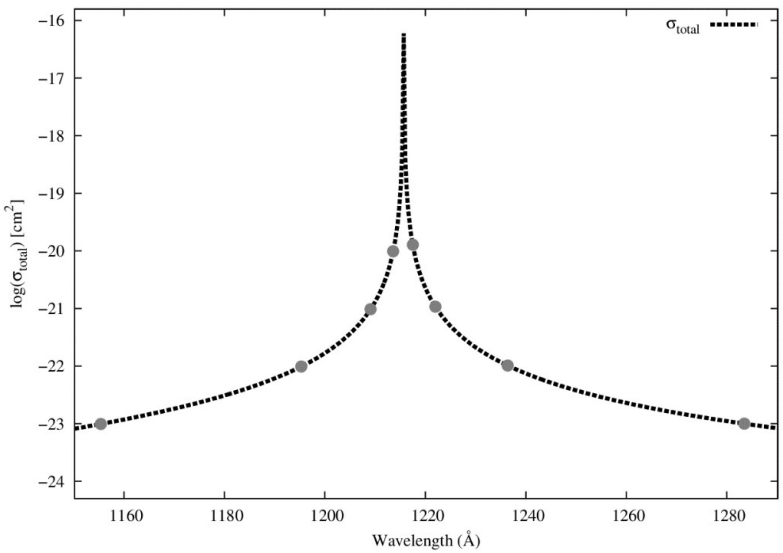
$BR_3(\lambda)$: Branching ratio to level 3

$$\lambda = \lambda_{min} + (\lambda_{max} - \lambda_{min}) * iwl/iwlmax$$

$$\mathbf{intp} = \mathbf{nint}(f_\lambda) \qquad \boldsymbol{\tau_z = \sigma_{total} N_{HI}}$$



Cross section and branching ratio of Lyβ and Lyγ (Chang et al. 2015)



Cross section of Lyα

Monte Carlo Simulation

1. Generate the initial UV photon

$\mu = 2 * ran2(idum) - 1 \qquad (= \cos\theta)$

$\varphi = 2 * pi * ran2(idum)$

$\rho_{11} = 0.5 \quad \rho_{21} = 0 \quad \rho_{22} = 0.5$

2. Set the initial scattering position

$\tau = -\log(ran2(idum))$

$x = \tau * \cos(\varphi) * sqrt(1 - \mu^2)$

$y = \tau * \sin(\varphi) * sqrt(1 - \mu^2)$

$z = \tau * \mu$

3. Check weather position is inside or outside

4. Generate the next wavevector

$\mu' = 2 * ran2(idum) - 1 \qquad (= \cos\theta)$

$\varphi' = 2 * pi * ran2(idum)$

Density Matrix Transition

5. Check weather scattering is Raman or Rayleigh scattering.

$\tau = -\log(ran2(idum))$

$x' = x + \tau * \cos(\varphi) * sqrt(1 - \mu^2)$

$y' = y + \tau * \sin(\varphi) * sqrt(1 - \mu^2)$

$z' = z + \tau * \mu$

6. Check weather position is inside or outside

$x = x' \quad y = y' \quad z = z'$

go to 4.

Raman scattered photon

$imu = int(10 * mu + 11)$

$tfluxn(iwl) = tfluxn(iwl) + 1$

$fluxn(imu, iwl) = fluxn(imu, iwl) + 1$

$poln(imu, iwl) = poln(imu, iwl) + den11 - den22$

Rayleigh scattered photon

$imu = int(10 * mu + 11)$

$tflux1(iwl) = tflux1(iwl) + 1$

$flux1(imu, iwl) = flux1(imu, iwl) + 1$

$pol1(imu, iwl) = pol1(imu, iwl) + den11 - den22$